

Name _____ Per _____

LO: I can connect right triangles to the equation of a circle, write equations of circles, and solve problems involving circles and the coordinate plane.

DO NOW On the back of this packet

(1) **What do right triangles have to do with the equation of a circle?**

calculator

On a graph, circles are defined by their **radius** and **center**. To write an equation for a circle, we need to find a relationship between the center, the radius, and the x and y coordinates. Let's start by looking at a circle with a radius of 5 and center at the origin $(0,0)$. This circle is graphed on the do now page called "The Circle". (You can tear "The Circle" page off to make your work easier.)

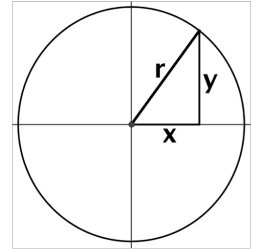
- Highlight the shape formed by the x value, the y value, and the **radius** of the circle that is drawn to point A.
- The shape you highlighted is a _____.
- The radius of the circle is also the _____ of the _____.
- The x value for the point on the circle is also the length of a _____ of the right triangle. } leg or
- The y value for the point on the circle is also the length of a _____ of the right triangle. } hypotenuse
- The equation for relating lengths of a right triangle is _____.
- For each point on the circle, write an equation by plugging in the x , y , and r values and verifying that the equation works by calculating and checking. If the equation works for graph, circle the letter.

A**E****I****B****F****J****C****G****K****D****H****L**

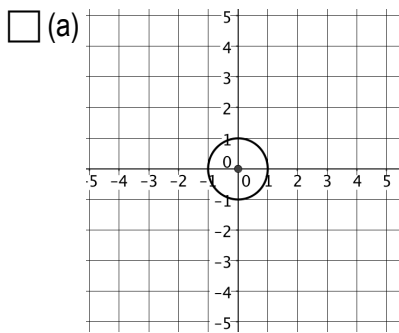
(2)
calculator

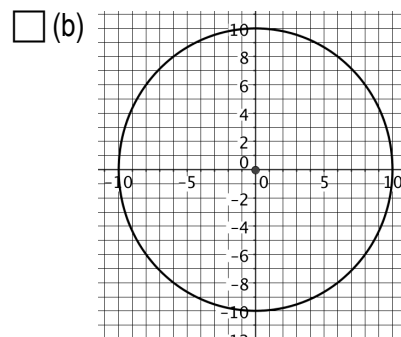
Writing an equation for a circle centered at the origin

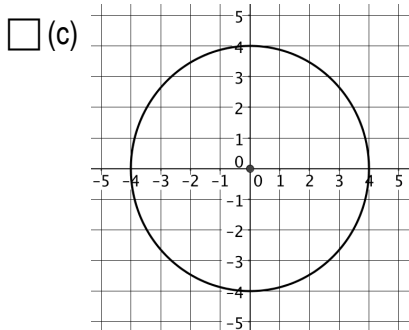
- (a) Did the equation work for all 12 points of the circle? _____ Will it work for every point on the circle? _____
 (b) Use the diagram at right to write a general equation for circles centered at the origin:

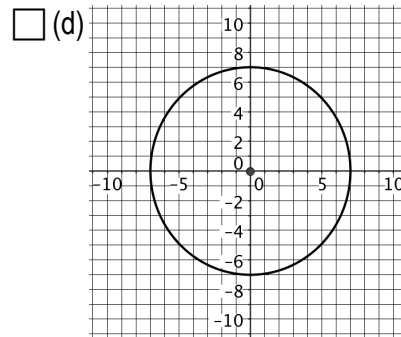


Write an equation with the variables x and y for each circle. The center of each circle is the origin.









(e) radius = 20

(f) radius = 11

(g) radius = 3

(h) radius = 9

(You may want to sketch i and j to see the circle first.)

(i) $(0,2)$ is a point on the circle

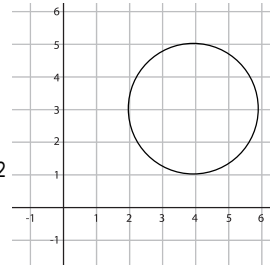
(j) $(-8, 0)$ is a point on the circle

(3)
calculator

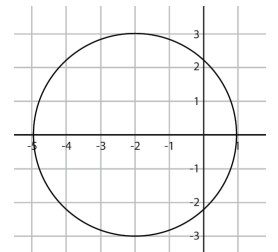
Writing an equation for a circle NOT centered at the origin

What if the center is NOT at the origin? We could write the equation for the circle if the center coordinates were (0,0). So, how can we “drag” or **translate** the center of the circle back to the origin?

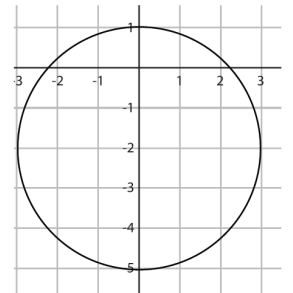
The coordinates for the circle center are: (____, ____)
 Translate x & y so the center is back at the origin (____, ____)
 What will our equation look like? (____)² + (____)² = (____)²
 Simplify the equation _____



The coordinates for the circle center are: (____, ____)
 Translate x & y so the center is back at the origin (____, ____)
 What will our equation look like? (____)² + (____)² = (____)²
 Simplify the equation _____



The coordinates for the circle center are: (____, ____)
 Translate x & y so the center is back at the origin (____, ____)
 What will our equation look like? (____)² + (____)² = (____)²
 Simplify the equation _____

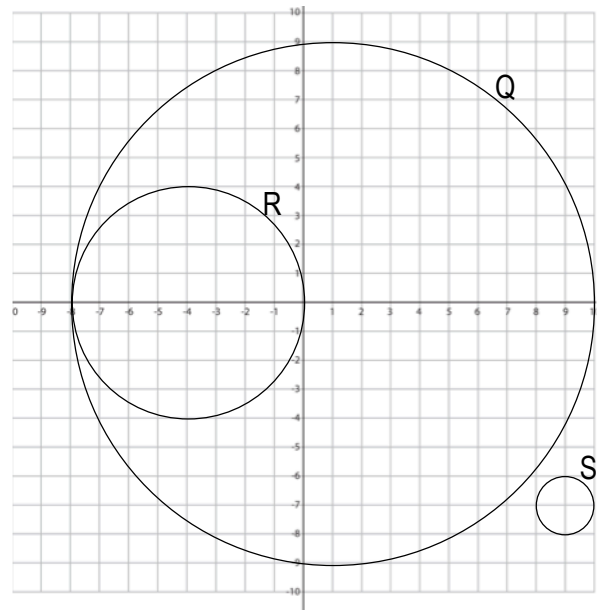


Write an equation for each circle:

Q:

R:

S:



(4)
calculator

Center and radius from equations and equations from center and radius

We know that equations for circles can be written $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius. We have written equations when we know the center and radius by translating the center back to the origin and figuring out the distance from the center of the circle to a point on the circle. What if we know the equation, and have to figure out the center and radius?

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y + 5)^2 = 4$$

Center _____ (HINT, this means what point did we have to translate the center from?)

Radius _____ (HINT, r^2 is what you see in the equation. What do you have to do to find r ?)

Which equation represents the circle whose center is $(-2, 3)$ and whose radius is 5?

1) $(x - 2)^2 + (y + 3)^2 = 5$ 2) $(x + 2)^2 + (y - 3)^2 = 5$ 3) $(x + 2)^2 + (y - 3)^2 = 25$ 4) $(x - 2)^2 + (y + 3)^2 = 25$

What is an equation of a circle with center $(7, -3)$ and radius 4?

1) $(x - 7)^2 + (y + 3)^2 = 4$ 2) $(x + 7)^2 + (y - 3)^2 = 4$ 3) $(x - 7)^2 + (y + 3)^2 = 16$ 4) $(x + 7)^2 + (y - 3)^2 = 16$

If we know the endpoints for the diameter, we can figure out what we need to write the equation. To write the equation of a circle we need the _____ and the _____. To help get your mind around the problem, consider:

Is the center ALWAYS on the diameter? _____

When it is on the diameter, where is it located? _____

Once you find the center point, how can you use an endpoint of the diameter to figure out the radius?

(4) Write an equation of the circle whose diameter AB has endpoints $A(5, 12)$ and $B(-5, 12)$.

Center: _____ Radius: _____ Equation: _____

(5) Each pair of points are the endpoints of the diameter of a circle. For each circle, find the center and radius and write an equation. SHOW YOUR WORK

a) $(-2, 3)$ and $(6, 3)$

b) $(5, 12)$ and $(-5, 12)$

c) $(2, 5)$ and $(2, 13)$

(5) Using diameter endpoints to write an equation of a circle
calculator

Answer each question below:

- 5 Which equation represents the circle whose center is $(-2, 3)$ and whose radius is 5?

- 1) $(x - 2)^2 + (y + 3)^2 = 5$
- 2) $(x + 2)^2 + (y - 3)^2 = 5$
- 3) $(x + 2)^2 + (y - 3)^2 = 25$
- 4) $(x - 2)^2 + (y + 3)^2 = 25$

- 3 The center and radius of the given circle

$(x - 3)^2 + (x + 8)^2 = 39$ are:

- 1) $(3, -8), r = 39$
- 2) $(-3, -8), r = \sqrt{39}$
- 3) $(-3, 8), r = \sqrt{39}$
- 4) $(3, -8), r = \sqrt{39}$

- 9 A circle is represented by the equation $x^2 + (y + 3)^2 = 13$. What are the coordinates of the center of the circle and the length of the radius?

- 1) $(0, 3)$ and 13
- 2) $(0, 3)$ and $\sqrt{13}$
- 3) $(0, -3)$ and 13
- 4) $(0, -3)$ and $\sqrt{13}$

- 1 What are the center and radius of a circle whose equation is $(x - A)^2 + (y - B)^2 = C$?

- 1) center = (A, B) ; radius = C
- 2) center = $(-A, -B)$; radius = C
- 3) center = (A, B) ; radius = \sqrt{C}
- 4) center = $(-A, -B)$; radius = \sqrt{C}

- 9 The center of a circular sunflower with a diameter of 4 centimeters is $(-2, 1)$. Which equation represents the sunflower?

- 1) $(x - 2)^2 + (y + 1)^2 = 2$
- 2) $(x + 2)^2 + (y - 1)^2 = 4$
- 3) $(x - 2)^2 + (y - 1)^2 = 4$
- 4) $(x + 2)^2 + (y - 1)^2 = 2$

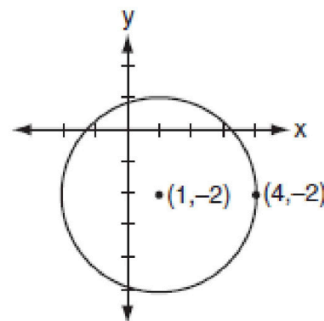
- 10 The center of a circle represented by the equation $(x - 2)^2 + (y + 3)^2 = 100$ is located in Quadrant

- 1) I
- 2) II
- 3) III
- 4) IV

- 3 Which equation represents circle O with center $(2, -8)$ and radius 9?

- 1) $(x + 2)^2 + (y - 8)^2 = 9$
- 2) $(x - 2)^2 + (y + 8)^2 = 9$
- 3) $(x + 2)^2 + (y - 8)^2 = 81$
- 4) $(x - 2)^2 + (y + 8)^2 = 81$

- 5 Which equation represents the circle shown in the accompanying graph?



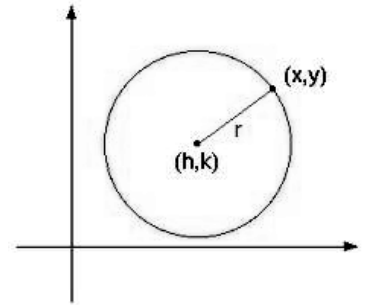
- 1) $(x - 1)^2 - (y + 2)^2 = 9$
- 2) $(x - 1)^2 + (y + 2)^2 = 9$
- 3) $(x + 1)^2 - (y - 2)^2 = 9$
- 4) $(x + 1)^2 + (y - 2)^2 = 9$

(6)
calculator

Circle Summary

SUMMARIZE: For any circle with center (h, k) and radius r , the equation of the circle can be written:

where _____ is the center and _____ is the radius



If I know the endpoints of a diameter of a circle, I can use them to find the:

center by finding _____

radius by finding the distance from _____ to _____

OR by finding half the distance from one _____ to _____

(7)
calculator

Completing the Square to write equations of circles in center-radius form

Steps to verify the equation of a circle by completing the square:

1. Isolate the constant on one side of the equation and group all x terms together and group all y terms together.
2. Take one-half the coefficient of x and square it. Also, take one-half the coefficient of y and square it. Namely,

$$\left(\frac{1}{2} \cdot \text{coeff of } x\right)^2 \quad \text{and} \quad \left(\frac{1}{2} \cdot \text{coeff of } y\right)^2$$

3. Add the result of each part of step 2 to both sides of the equation.
4. Factor as two perfect squares.

Example 1

The following is the equation of a circle with radius 5 and center $(1, 2)$. Do you see why?

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$$

(7) **Completing the Square to write equations of circles in center-radius form**

cont.

Exercise

1. Rewrite the following equations in the form $(x - a)^2 + (y - b)^2 = r^2$.

a. $x^2 + 4x + 4 + y^2 - 6x + 9 = 36$

b. $x^2 - 10x + 25 + y^2 + 14y + 49 = 4$

(8)
calculator

Completing the Square to write equations of circles in center-radius form

Example 2

What is the center and radius of the following circle?

$$x^2 + 4x + y^2 - 12y = 41$$

Exercises

2. Identify the center and radius for each of the following circles.

a. $x^2 - 20x + y^2 + 6y = 35$

b. $x^2 - 3x + y^2 - 5y = \frac{19}{2}$

**Completing the Square to write equations of circles in center-radius form**

Answer each question below:

Problem Set

- Identify the center and radii of the following circles.
 - $(x + 25) + y^2 = 1$
 - $x^2 + 2x + y^2 - 8y = 8$
 - $x^2 - 20x + y^2 - 10y + 25 = 0$
 - $x^2 + y^2 = 19$
 - $x^2 + x + y^2 + y = -\frac{1}{4}$
- Sketch a graph of the following equations.
 - $x^2 + y^2 + 10x - 4y + 33 = 0$
 - $x^2 + y^2 + 14x - 16y + 104 = 0$
 - $x^2 + y^2 + 4x - 10y + 29 = 0$
- Chante claims that two circles given by $(x + 2)^2 + (y - 4)^2 = 49$ and $x^2 + y^2 - 6x + 16y + 37 = 0$ are externally tangent. She is right. Show that she is.
- Draw a circle. Randomly select a point in the interior of the circle; label the point A . Construct the greatest radius circle with center A that lies within the circular region defined by the original circle. Hint: Draw a line through the center, the circle, and point A .

(5) What are the center and radius of the circle with the equation: $(x^2 + 2x + 1) + (y^2 + 4y + 4) = 121$.

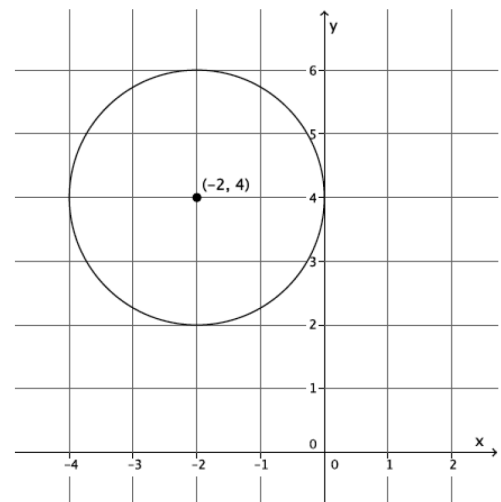
(10) **Exit Ticket**
calculator

ON THE LAST PAGE

(11) **Homework**
calculator

3. Write an equation for the circle whose center is at $(9, 0)$ and has radius 7.

4. Write an equation whose graph is the circle below.



5. What is the radius and center of the circle given by the equation $(x + 12)^2 + (y - 4)^2 = 81$?

6. Petra is given the equation $(x - 15)^2 + (y + 4)^2 = 100$ and identifies its graph as a circle whose center and radius is 10. Has Petra made a mistake? Explain.

(11) Homework
calculator

7.

a. What is the radius of the circle with center $(3, 10)$ that passes through $(12, 12)$?

b. What is the equation of this circle?

8. A circle with center $(2, -5)$ is tangent to the x -axis.

a. What is the radius of the circle?

b. What is the equation of the circle?

9. Two points in the plane, $A = (-3, 8)$ and $B = (17, 8)$, represent the endpoints of the diameter of a circle.

a. What is the center of the circle? Explain.

b. What is the radius of the circle? Explain.

c. Write the equation of the circle.

(11) Homework
calculator

10. Consider the circles with equations:

$$x^2 + y^2 = 25 \text{ and}$$
$$(x - 9)^2 + (y - 12)^2 = 100.$$

- What are the radii of the circles?
 - What is the distance between the centers of the circles?
 - Make a rough sketch of the two circles to explain why the circles must be tangent to one another.
11. A circle is given by the equation $(x^2 + 2x + 1) + (y^2 + 4y + 4) = 121$.
- What is the center of the circle?
 - What is the radius of the circle?
 - Describe what you had to do in order to determine the center and the radius of the circle.

(11) Homework

calculator

1. Write the equation for a circle with center $\left(\frac{1}{2}, \frac{3}{7}\right)$ and radius $\sqrt{13}$.

2. What is the center and radius of the circle given by the equation $x^2 + (y - 11)^2 = 144$?

3. A circle is given by the equation $x^2 + y^2 = 100$. Which of the following points are on the circle?
 - a. $(0, 10)$
 - b. $(-8, 6)$
 - c. $(-10, -10)$
 - d. $(45, 55)$
 - e. $(-10, 0)$

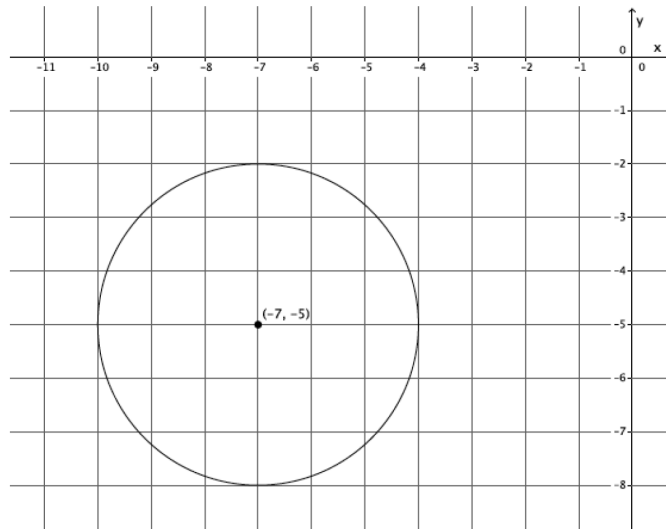
4. Determine the center and radius of each circle:
 - a. $3x^2 + 3y^2 = 75$
 - b. $2(x + 1)^2 + 2(y + 2)^2 = 10$
 - c. $4(x - 2)^2 + 4(y - 9)^2 - 64 = 0$

5. A circle has center $(-13, \pi)$ and passes through the point $(2, \pi)$.
 - a. What is the radius of the circle?
 - b. Write the equation of the circle.

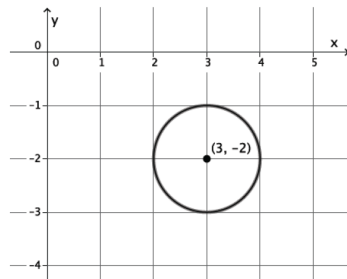
(11) Homework
calculator

6. Two points in the plane, $A = (19, 4)$ and $B = (19, -6)$, represent the endpoints of the diameter of a circle.
- What is the center of the circle?
 - What is the radius of the circle?
 - Write the equation of the circle.

7. Write the equation of the circle shown below.



8. Write the equation of the circle shown below.



9. Consider the circles with equations:

$$x^2 + y^2 = 2 \text{ and}$$

$$(x - 3)^2 + (y - 3)^2 = 32.$$

- What are the radii of the two circles?
- What is the distance between their centers?
- Make a rough sketch of the two circles to explain why the circles must be tangent to one another.

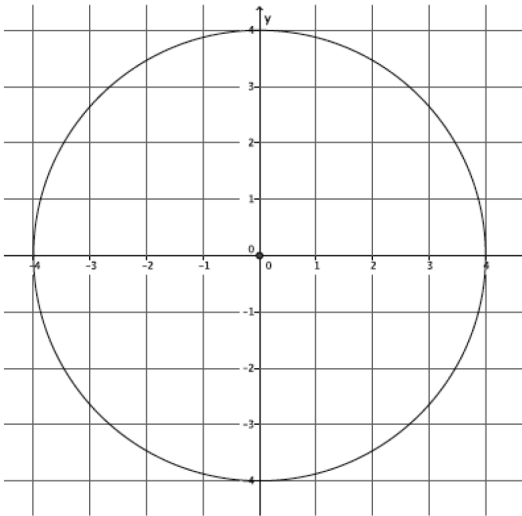
Exit Ticket Name _____ Date _____ Per _____ 8.5R

(1) The LO (Learning Outcomes) are written below your name on the front of this packet. Demonstrate your achievement of these outcomes by doing the following:

1. Describe the circle given by the equation $(x - 7)^2 + (y - 8)^2 = 9$.

2. Write the equation for a circle with center $(0, -4)$ and radius 8.

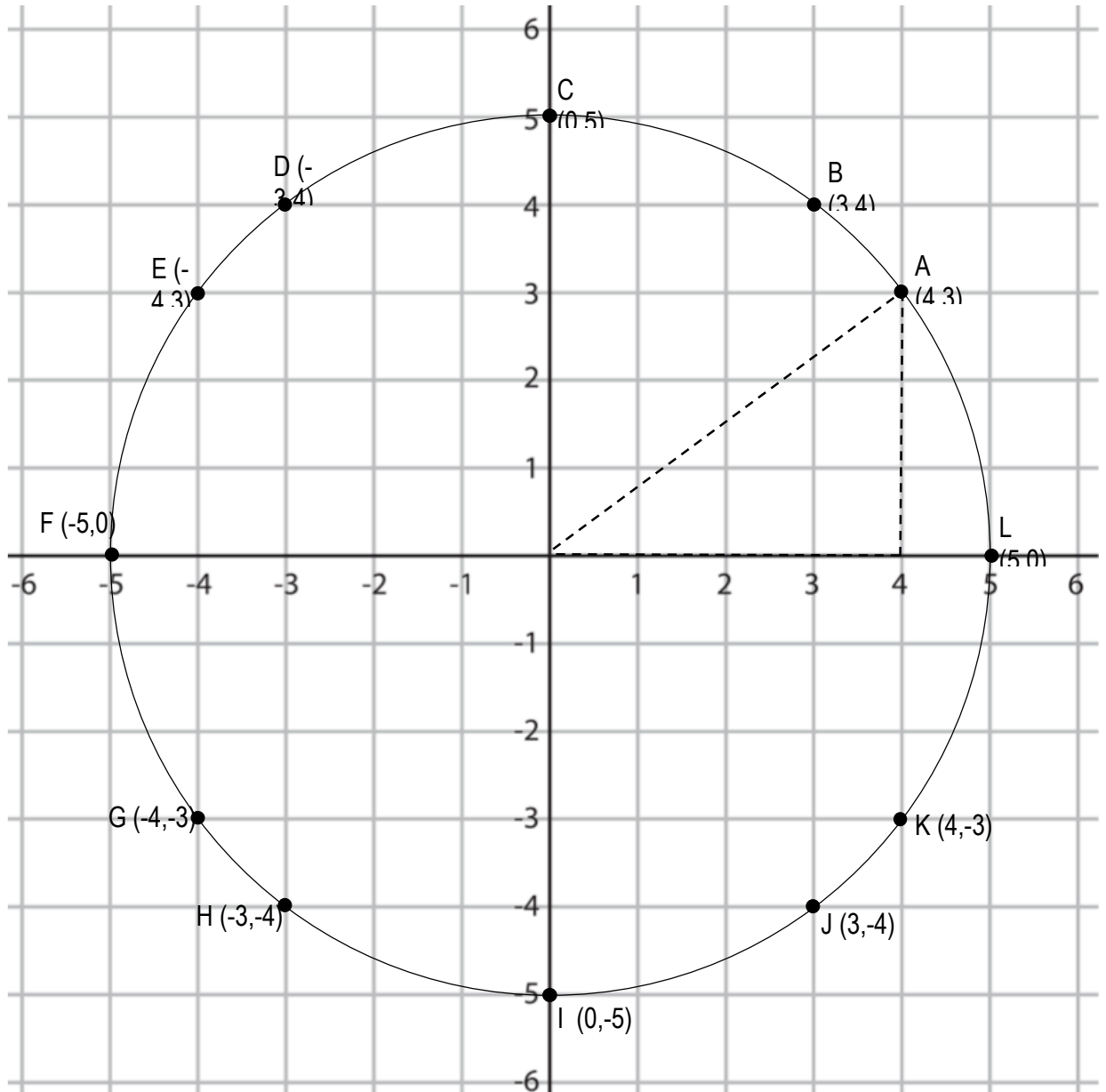
3. Write the equation for the circle shown below.



4. A circle has a diameter with endpoints at $(6, 5)$ and $(8, 5)$. Write the equation for the circle.

5. Write the standard form equation for the circle: $8x + x^2 - 2y = 64 - y^2$

(1) Use the circle at right to do problem #1 in your packet. You may want to use highlighter. You may want to tear this sheet off so you are not flipping back and forth.



(2) What about the cartoon below is supposed to make people smile?

